① Evaluate 1^2 -2^2 +3^2 -4^2 +5^2 -6^2 + ... + 99^2 -100^2

$\rightarrow \quad 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \cdots + 99^2 - 100^2$

$1^2 = 1 \times 1 = 1$

$2^2 = 2 \times 2 = 4$

$3^2 = 3 \times 3 = 9$

$1^2 - 2^2 = (1-2)(1+2)$

$1 - 4 = (-1)(3)$

$\parallel$

-3

$a^2 - b^2 = (a-b)(a+b)$

$a^2 + \cancel{ab} - \cancel{ab} - b^2$

$= a^2 - b^2$

$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \cdots + 99^2 - 100^2$

$= (1-2)(1+2) + (3-4)(3+4) + (5-6)(5+6) + \cdots +$

$(99-100)(99+100)$

$= (-1)(1+2) + (-1)(3+4) + (-1)(5+6) + \cdots +$

$(-1)(99+100)$

$= (-1)(1 + 2 + 3 + 4 + \cdots + 99 + 100)$

\# $1+2+\cdots+n = \boxed{\dfrac{n(n+1)}{2}}$

Proof :-

$S = 1+2+\cdots+n$

$$
\begin{aligned}
2S = \;& 1 + 2 + \cdots + n \\
+ \;& n + (n-1) + \cdots + 1 \\
\hline
& (n+1) + (n+1) + \cdots + (n+1)
\end{aligned}
$$

$2S = n(n+1)$

$S = \dfrac{n(n+1)}{2}$

Our answer is

$= (-1) \times \left(\dfrac{100 \times 101}{2} \right)$

$= (-1)\, 5050$

$= -5050$

Q. What are the two natural numbers whose difference is 66 and the least common multiple is 360.

Solution:- (Vihaan's method)

Factors of 360 = 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 24, 30, 36, 40, 45, 72, 90, 120, 180, 360

Just bash :(

Solution:-

let one number be x. let the other number be y.

So $x - y = 66$, $LCM(x, y) = 360$

$$\boxed{x \cdot y = LCM(x, y) \cdot HCF(x, y)}$$

$$HCF(x, y) = HCF(x, y - x)$$

$$HCF(10, 50) = HCF(10, 40) = HCF(10, 30)$$

$$= HCF(10, 20) = HCF(10, 10) = 10$$

$$HCF(x, y) = HCF(x, y - x) = HCF(x, 66)$$

$HCF(x, y) \mid x$ and $HCF(x, y) \mid y$.

$HCF(x,y) = HCF(x, y-x) = HCF(x,66) \mid 66$

$66 = 6 \times 11$

Also LCM of $x,y = 360 = 6 \times 360$

So $HCF(x,y) = 6$

$x - y = 66$ — (1)

$x \cdot y = 360 \times 6 = 2160$

$(x+y)^2 = (x-y)^2 + 4xy = 66^2 + 4 \times 2160$

$= 114^2$

So $x + y = 114$ — (11)

So $2x = 114 + 66 = 180$

$\Rightarrow x = 90$ $\Rightarrow y = 24$

Q: Find the largest number which would divide 50 and 60 leaving remainders 8 and 4 respectively.

→ let the number be x.

$$x \overline{)\,50\,}$$
$$-$$
$$8$$

$$x \overline{)\,60\,}$$
$$-$$
$$4$$

$x \mid 50 - 8$ and $x \mid 60 - 4$

$x \mid 42$ and $x \mid 56$

So $HCF(42, 56) = 14$