Q. Find the least number which on being divided by 5, 6, 8, 9, 12 leaves in each case a remainder 1 but when divided by 13 leaves no remainder.

Q. (b) Find the sum of all the digits of the results of the subtraction $10^{99} - 99$.

Rough:-

$10^{99} - 99 \rightarrow \underbrace{1000\cdots0}_{99 \text{ zeroes}}$

$10^1 = 10$

$10^2 = 10 \times 10 = 1\underbrace{00}_{2 \text{ zeros}}$

$10^{99} = \underbrace{10 \times 10 \times 10}_{99} \cdots$

$10^3 = 10 \times 10 \times 10 = 1\underbrace{000}_{3}$

$$\begin{array}{r} 1000\cdots00 \\ -\quad 99 \\ \hline \underbrace{999\cdots9}_{97}11 \end{array}$$

Q. let the number be x.

So $x-1$ is divisible by 5, 6, 8, 9, 12.

Hence $\text{LCM}(5,6,8,9,12) \mid x-1$.

So the $\text{LCM}(5,6,8,9,12) = 360 \mid x-1 \Rightarrow x-1 = 360k$

So the number $x = 360k + 1$

Now, $x = 360k+1$ is divisible by 13.

So we need to find the smallest k such that

$13 \mid x = 360k+1$

$\rightarrow$ $k=10$, is the smallest.

Q. (c) Find the units digit of the number $(4444)^{4444}$

Q.) A three digit number 4a3 is added to another three digit number 984 to give a four digit number 13b7, which is divisible by 11. Find the value of 2a+b.

$\hookrightarrow$ Given 13b7 is divisible by 11.

So $(3+7)-(1+b)$ is divisible by 11

$= 9-b$ is divisible by 11.

Since b is a digit, so $b = 0, 1, 2, \cdots, 9$.

So $\boxed{b=9}$

$4a3 + 984 = 13b7 = 1397$

$\Rightarrow 4a3 = 1397 - 984 = 413 \Rightarrow a=1$

So answer is $2a+b = 2\times1+9 = 11$

$$\begin{array}{r} 4a3 \\ +\ 984 \\ \hline 13b7 \end{array} \Rightarrow$$ So $a = 0, 1$

So if $a=0$, $b=8$, but $13b7 = 1387$ is not divisible by 11

So $a=1, b=9$

Q. (c) Find the units digit of the number $(4444)^{4444}$

$\rightarrow (4444)^{4444} = \underbrace{4444 \times 4444 \times 4444 \times \cdots}_{4444}$

Unit digit of 4444^{4444} is same as 4^{4444}.

$\searrow 6$

$4^1 \rightarrow 4$

$4^2 \rightarrow 4 \times 4 = 16 \rightarrow 6$

$4^3 \rightarrow 64 \rightarrow 4$

$4^4 \rightarrow 4 \times 4 \times 4 \times 4 = 256 \rightarrow 6$

$4^5 \rightarrow 4$

$4^6 \rightarrow 6$

$$\boxed{\begin{array}{l} 4^{even} \rightarrow 6 \\ 4^{odd} \rightarrow 4 \end{array}}$$