

① Solve the following equation for integer values of x and y . $(2x+y)(5x+3y) = 7$

Solution:- So if x & y are integers, then so is $2x+y$ & $5x+3y$

$$(2x+y)(5x+3y) = 7$$

$$7 = 1 \times 7$$

$$7 = -1 \times -7$$

So $7 = 1 \times 7$

Case 1

$$(2x+y) = 1, (5x+3y) = 7$$

Case 2

$$(2x+y) = 7, (5x+3y) = 1$$

Case 3

$$(2x+y) = -1, (5x+3y) = -7$$

Case 4

$$(2x+y) = -7, (5x+3y) = -1$$

Case 1 $(2x+y) = 1$, $(5x+3y) = 7$

$$2x+y=1 \quad - \textcircled{i}$$

$$5x+3y=7 \quad - \textcircled{ii}$$

$$\hookrightarrow \textcircled{i} \times 3$$

$$\Rightarrow (2x+y) \times 3 = 1 \times 3$$

$$6x+3y=3 \quad - \textcircled{iii}$$

$$\textcircled{iii} - \textcircled{ii}$$

$$6x+3y=3$$

$$- (5x+3y)=7$$

$$6x-5x = 3-7$$

$$\Rightarrow x = -4$$

$$x = -4, y = 9$$

$$(2x+y) \times 5 = 5$$

$$(5x+3y) \times 2 = 14$$

$$\Rightarrow 10x+5y=5$$
$$10x+6y=14$$

$$10x+6y=14$$

$$- (10x+5y)=5$$

$$y = 9$$

Case 2

$$(2x + y) = 7, \quad (5x + 3y) = 1$$

Solution: -

$$(2x + y) = 7 \quad \text{--- (i)}$$

$$(5x + 3y) = 1 \quad \text{--- (ii)}$$

$$\text{(i) } \times 3, \quad 6x + 3y = 21 \quad \text{--- (iii)}$$

$$\text{(iii)} - \text{(ii)} \Rightarrow \quad 6x + 3y = 21$$

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$$5x + 3y = 1$$

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$$6x + 3y - (5x + 3y) = 21 - 1$$

$$\Rightarrow x = 20$$

$$\Rightarrow 2x + y = 7 \quad \& \quad x = 20, \quad 40 + y = -7 \quad \Rightarrow y = -33$$

Case 3

$$(2x+y) = -1, (5x+3y) = -7$$

$$2x+y = -1 \quad - \textcircled{i}$$

$$5x+3y = -7 \quad - \textcircled{ii}$$

$$\textcircled{i} \times 3 \Rightarrow 6x+3y = -3 \quad - \textcircled{iii}$$

$$\begin{aligned} \textcircled{iii} - \textcircled{ii} &\Rightarrow 6x+3y - (5x+3y) = -3 - (-7) \\ &\Rightarrow x = 4 \end{aligned}$$

$$y = -9 \quad \left| \begin{array}{l} 2 \times 4 + y = -1 \\ \Rightarrow 8 + y = -1 \\ \Rightarrow y = -9 \end{array} \right.$$

Case 4

$$(2x+y) = -7, (5x+3y) = -1$$

Solution:- $(2x+y) = -7 \quad - \textcircled{i}$

$$(5x+3y) = -1 \quad - \textcircled{ii}$$

$$\textcircled{i} \times 3 \Rightarrow 6x+3y = -21 \quad - \textcircled{iii}$$

$$\textcircled{\text{ii}} - \textcircled{\text{ii}} = x = -21 - (-1) = -20$$

$$\text{So } 2x(-20) + y = -7$$

$$\Rightarrow y = 33 .$$