

Problems to try part 2

The philomath club

- ① Find the ratio of the least and the greatest four digit numbers which are exactly divisible by 34.

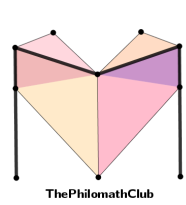
Ratio of two numbers a and b ; $a:b$, $\frac{a}{b}$

For example ratio of 15 and 10 = $15:10 = \frac{15}{10} = \frac{3}{2} = 3:2$

- ② The price of sugar being raised 50%, by how much percent must a man reduce his consumption of that article so as not to increase his expenditure?

- ③ 90% of the boys of a school pass in English and 85% in Arithmetic, 150 boys pass in both the subjects and no boy fails in both. How many boys are there in the school?

- ④ Find the digit values of x and y in order that the number $5x16y5$ may be divisible by 275.



1

Find the ratio of the least and the greatest four digit numbers which are exactly divisible by 34.

Ans :- smallest 4 digit number = 1000

$$1000 = 29 \times 34 + 14$$

so the least four digit number divisible by 34 =

$$1000 - 14 + 34 = 30 \times 34 = 1020$$

largest 4 digit number = 9999

$$9999 = 294 \times 34 + 3$$

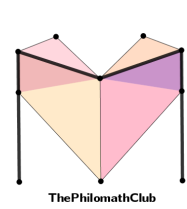
so the largest 4 digit divisible by 34 = $9999 - 3$

$$= 9996 = 34 \times 294$$

So the ratio = $1020 : 9996 = 30 \times 34 : 294 \times 34$

$$= 30 : 294$$

$$= 5 : 49$$



(2)

The price of sugar being raised 50%, by how much percent must a man reduce his consumption of that article so as not to increase his expenditure ?

$$\text{Answer: } 33 \frac{1}{3} \% = 33.333 \dots \% = 33.\overline{3} \%$$

Solution:- Suppose the price of sugar = 100

When the price is being raised by 50% = $100 + 50 = 150$

$$\frac{150 - 100}{150} \% = \frac{50}{150} \% = 33 \frac{1}{3} \%$$

Original solution:- Let price of sugar be X . And the monthly consumption of sugar be Y .

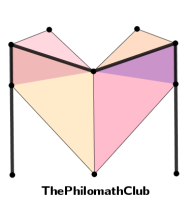
Since, the price of sugar is raised by 50%/-

$$\text{New price of sugar is } \frac{50}{100} \times X + X = \frac{3X}{2}$$

The man buys Y amount of sugar of price X ,

$$\text{Total money} = XY$$

New price of sugar is $\frac{3X}{2}$, so new amount of sugar (monthly) = $\frac{2}{3} Y$.

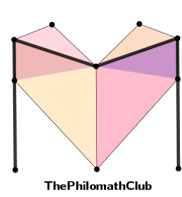


Originally, the consumption was Y .

Now, he buys $\frac{2}{3} Y$ amount of sugar

$$\text{So the reduction (\%)} = \frac{Y - \frac{2}{3} Y}{Y} \times 100\% = \frac{\frac{1}{3} Y}{Y} \times 100\% = \frac{1}{3} \cdot 100\% \times 100$$
$$= \frac{100}{3} = 33.\bar{3}\%$$

$$\text{reduction \%} = \frac{\text{original price} - \text{new price}}{\text{original price}}$$



3

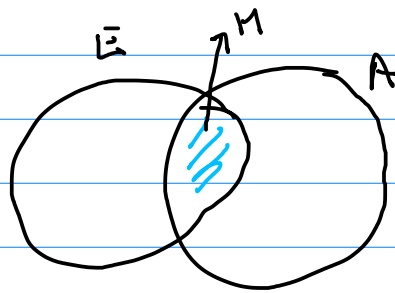
90% of the boys of a school pass in English and 85% in Arithmetic, 150 boys pass in both the subjects and no boy fails in both. How many boys are there in the school?

let the total number of boys be x .

let E denote the ^{no of} boys who passed in English

let A denote the no of boy who passed in arithmetic

let M denote the no of boys who passed in both

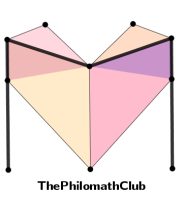


$$\begin{aligned} \text{So basically } x &= E + A - M \\ &= \frac{90 \cdot x}{100} + \frac{85 \cdot x}{100} - 150 \end{aligned}$$

$$\begin{aligned} 100x &= 90x + 85x - 15000 \\ &= 175x - 15000 \end{aligned}$$

$$15000 = 75x$$

$$\Rightarrow x = 200$$



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(4)

Find the digit values of x and y in order that the number $5x16y5$ may be divisible by 275 .

Solution :- $275 = 25 \times 11$

Thus $5x16y5$ is divisible by 275

$\Rightarrow 5x16y5$ is divisible by both 25 and 11 .

but the $5x16y5$ is divisible by 11

if $5+1+y - (x+6+5)$ is divisible by 11
 $= y-x-5$ is divisible by 11

Test divisibility of 25 .

$$5x16y5 = 5x1600 + y5$$

So $25 \mid y5 \Rightarrow y = 2, 7$

Now we have $y-x-5$ divisible by 11

if $y = 2$, then $x = 8 \rightarrow 581625 \div 275 = 2115$

if $y = 7$, then $x = 2 \rightarrow 521675 \div 275 = 1897$