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① Suppose the least values obtained by two quadratics $x^2+3x+19$ and x^2-7x+3 (with x being Real) is a and b.

Find $a-b$.

Polynomials :- It is an expression composed variables and constants.

variable

$x+2$, x^2+3, x^2+y^3+4

constant

$x \times x + 3$ $\quad$ $x \times x + y \times y \times y + 4$

Ex

let $f(x) = x+2$

$f(1) = 1+2 = 3$

$f(3) = x+2$, when $x=3$

$= 3+2 = 5$

$f(3) = x+2$ at $x=3$
$= 5$

1 variable polynomial

Ex:- $f(x) = x^2+3$

$f(1) = 1^2+3 = 4$

$f(2) = 2^2+3 = 7$

$f(5) = 5^2+3 = 28$

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2 variable polynomial

Ex:- $f(x, y) = x^2 + y^3 + 4$

$f(1, 2) = 1^2 + 2^3 + 4 = 13$

Ex: $f(x) = 2x + 4$

variable

constant

$\boxed{2}x$

coefficient

$f(1) = 2 \times 1 + 4 = 6$

$f(3) = 2 \times 3 + 4 = 10$

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distributive Property

$(a + b)(x + y) = ax + ay + bx + by$

Ex:-

$(2 + 5)(7 + 3) = 7 \times 10 = 70$

$a = 2,\ b = 5,\ x = 7,\ y = 3$

$2 \times 7 + 2 \times 3 + 5 \times 7 + 5 \times 3 = 14 + 6 + 35 + 15$

$= 20 + 50$

$= 70$

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$$(a+b)^2 = (a+b)(a+b)$$

$$= a \times a + a \times b + b \times a + b \times b$$

$$= a^2 + ab + ba + b^2$$

$$= a^2 + 2ab + b^2$$

$$(a+b)^3 = [(a+b)(a+b)](a+b)$$

$$= [a^2 + 2ab + b^2](a+b)$$

$$= a^2 \times a + a^2 \times b + 2ab \times a + 2ab \times b + a \times b^2 + b^2 \times b$$

$$= a^3 + a^2 b + 2a^2 b + 2ab^2 + ab^2 + b^3$$

$$= a^3 + \overset{ab \times a}{3a^2 b} + \overset{ab \times b}{3ab^2} + b^3$$

$$= a^3 + 3ab(a+b) + b^3$$

$$3ab(a+b)$$

$$= 3ab \times a + 3ab \times b$$

$$= 3a^2 b + 3ab^2$$

$\therefore\ a^2 - b^2 = (a-b)(a+b)$

$\therefore\ (a-b)^2 = (a-b)(a-b)$

$= (a+(-b))(a+(-b))$

$(x+y)^2 = x^2 + 2xy + y^2$

take $x = a$, $y = -b$

$= a^2 + 2 \times a \times (-b) + (-b)^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$\therefore\ (a-b)(a+b) = a \times a + ab - ba - b \times b$

$= a^2 - b^2$

Completing the square method

$x^2 + 4x + 19 \longrightarrow$ complete the square

$\rightarrow$ square

$x^2 + 4x + 19$

$\|$

$x^2 + 2ax + a^2 + C$

$x^2 + 2ax + a^2$

So take $a = 2$

$x^2 + 4x + 19 = x^2 + 2 \times 2x + 2^2 + C$

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$x^2+4x+19 = x^2+4x+4+c$

So $c = 15$

$(x+a)^2+c$

$= (x+2)^2+15$

$x^2+4x+19 = (x+2)^2+15$

$x^2+6x+30 = x^2+2ax+a^2+c$

find a.

$= (x+3)^2+21$

This is useful in bounding quadratics

$x^2+4x+19 = (x+2)^2+15$

for all real x, $(x+2)^2 \geq 0$

$\cdot)$ for $x = -5$, $(-3)^2 = 9$

$(-5+2) = -3$, $(-5+2)^2 = (-3)^2 = 9$

Since $\boxed{(x+2)^2 \geq 0}$, we get that

$(x+2)^2+15 \geq 15$. $\rightarrow$ So $x^2+4x+19 \geq 15$.

So $x=-2$, $(x+2)^2=0$

So $(x+2)^2+15 = x^2+4x+19 = 15$

So the minimum value of

$x^2+4x+19$ is 15.

Ex: Find the minimum possible value of

$x^2+10x+26$ ove all real x.

For $x=0$, $0+0+26=26$

$x=-1$, $(-1)^2+10\times(-1)+26 = 1+(-10)+26 = 26-9$
$= 17$

$x^2+2ax+a^2+c = x^2+10x+26$

So $a=5$

So $x^2+10x+26 = (x+a)^2+c$
$= (x+5)^2+c$

So $c=1$

So $x^2+10x+26 = (x+5)^2+1$

Now $(x+5)^2 \geq 0$

So $(x+5)^2+1 \geq 1$. $\rightarrow$ checking $x=-5$
we get 1 as the minimum.