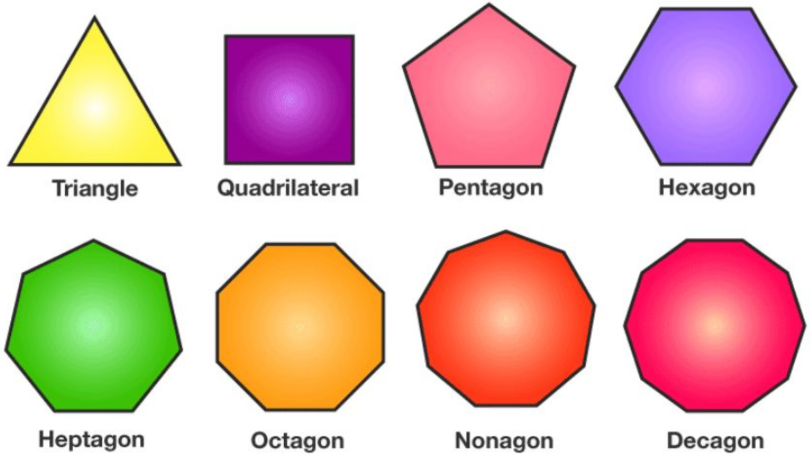
Polygons

The Philomath Club

What are polygons?

➢ Polygons can be found everywhere in our surroundings as well as in geometrical math. Many objects are in the shapes of polygons



Note: If polygon is n-sided, it is called an n-gon.

Some definitions!

Adjacent sides

Any two sides with a common end point are called the adjacent sides of the polygon.

Adjacent vertices

The end points of the same side of a polygon are called the adjacent vertices.

Diagonals

The line segments obtained by joining vertices which are not adjacent are called the diagonals of a polygon.

Concave polygon:

If a diagonal lies outside a polygon, then the polygon is called a concave polygon.

Convex polygon:

If all the diagonals lie inside the polygon, then the polygon is said to be a convex polygon.

Regular polygon:

A regular polygon is equiangular and equilateral. The word equiangular means, the interior angles of the polygon are equal to one another. The word equilateral means, the lengths of the sides are equal to one another.

Irregular polygon:

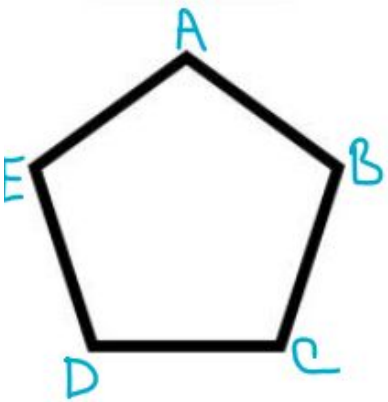
The polygon with unequal sides and unequal angles is called an irregular polygon.

Let’s count the number of diagonals in a polygon

- Let us take an n sided polygon.
- So to pick the first diagonal, we have n choices.
- Now the diagonal from that chosen point can go to (n-3) vertices only because sides don’t count as diagonals.
- But we counted every diagonal twice, so we divide by 2.
- Hence we have n(n-3)/2 diagonals in an n-gon

Let's see an example.

- Let us take n=5
- Now if we choose A as the first vertex, we cannot draw a diagonal to E and B because AE and AB are the sides of the pentagon.
- So already 2 vertices are gone and the one itself too.
- Hence we have (5-3)=2 (namely **D and C)** endpoints for the diagonal.
- Now from each vertex we draw a diagonal, we count that two times so we need to divide by two too.
- So we have 5(2)/2=5 diagonals in a pentagon.



Angle sum in a polygon.

- Angle sum of a polygon = (n - 2) x 180° , where n = number of sides of the polygon.
- A regular polygon has each angle equal to (2n-4)/n*90 degrees.
- The sum of the exterior angles of any polygon is 360°.
- The measure of each exterior angle in a regular polygon is 360°/n.

Proof of angle sum property in polygon.

- Now let’s choose a vertex in the polygon.
- We have (n-3) diagonals from that point to the vertices.
- These (n-3) vertices decompose the polygon into (n-2) triangles.
- So we have (n-2) non-overlapping triangles in the polygon.
- Hence the formula (n-2)*180 degrees of angle sum property.

  (Note: Try proving why exterior angle sum is 360 degrees).

Problems!

1) A polygon has 7 sides, how many diagonals are there in the polygon?

2) The interior angle of a regular polygon is 108 degrees. Find the number of sides in the polygon.

3) One angle of a pentagon is 140 degrees. If the remaining angles are in the ratio 1:2:3:4, what is the measure of the largest angle?

4) The difference between an exterior angle of (n-1) sided regular polygon and the exterior angle of (n+2) sided regular polygon is 6 degrees. Find the value of n.

5)The diagram shows the 10 exterior angles of a decagon. Find the sum of the exterior angles.

(You cannot assume the polygon is regular).

6) Find the number of sides in a regular polygon in which the interior angle is 172 degrees.

7) The sum of the interior angles of a polygon is three times the exterior angles. How many sides are there in the polygon?

8) ABCDEFGHIJKLMNOP is a regular 16-gon. Find $\angle ACB$ and $\angle ACD$.

9) If a polygon has 54 diagonals, find how many sides it has.

10) Challenge problem:

Let $ABCDE$ be a regular pentagon. If the internal bisectors of angles A and B meet at O, prove that OC, OD,OE also bisect angles C, D and E

Hint: Try using congruence and angle sum property to prove!

THANK YOU!