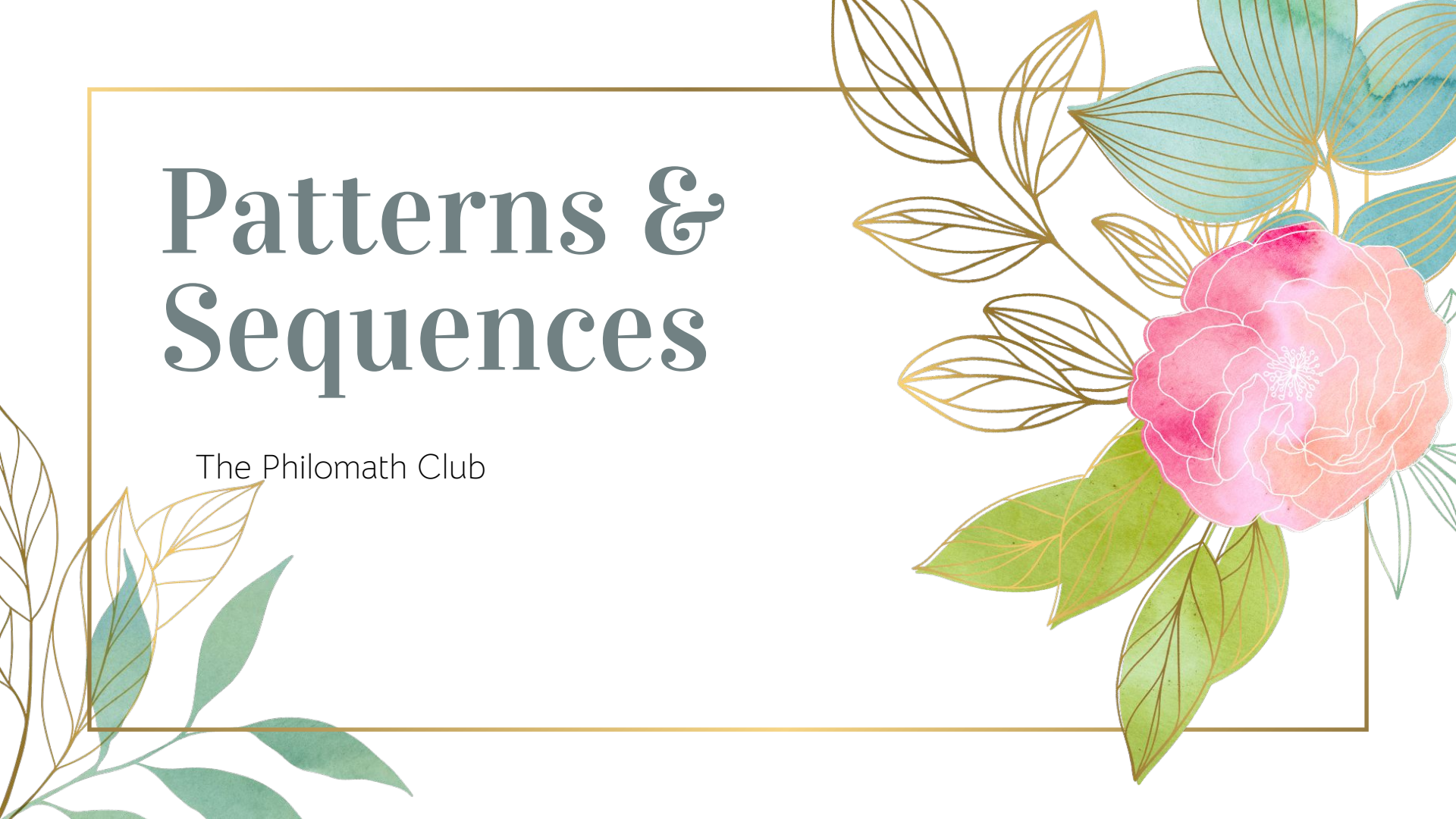
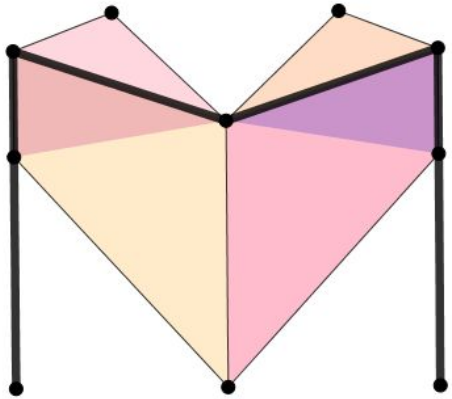
Patterns & Sequences

The Philomath Club





ThePhilomathClub

Hello!

I am here to teach you guys! I am in grade 11. Let's get into some problems!

Would you Rather?

The Philomath Club

Would you Rather?

have the ability to see 10 minutes into the future or 150 years into the future?

Would you Rather?

be forced to sing along or dance to every single song you hear?

Would you Rather?

swim in a pool full of Nutella or a pool full of maple syrup?

Would you Rather?

be 11 feet tall or nine inches tall?

Would you Rather?

win $25,000 or your best friend win $100,000

Ready?

If you get doubts please ask!

Bring out the inner Philomath!

Now we are ready!

Let’s start with the first set of slides

INTRO

1, 2, 3, 4, 5, 6, 7, ?

1, 3, 5, 7, 9, 11, ?

3, 6, 9, 12, 15, 18, 21, ?

Introduction

Guess the Pattern

- 1, 2, 3, 4, 5, 6, 7, ?
- 1, 3, 5, 7, 9, 11, ?
- 2, 4, 6, 8, 10, 12, 14, ?
- 3, 6, 9, 12, 15, 18, 21, ?
- 100, 200, 300, 400, 500, 600, 700, ?

Fibonacci is cool!

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Identify the pattern.

Solution : 1, 1, 2, 3, 5, 8, 13, 21, 34, . . .

Notice that

$1 + 1 = 2, \quad 1 + 2 = 3, \quad 2 + 3 = 5, \quad 3 + 5 = 8, \quad 5 + 8 = 13,$

$8 + 13 = 21, \quad 13 + 21 = 34, \quad 21 + 34 = 55, \quad 34 + 55 = 89, \ldots\ldots$

More about Fibonacci

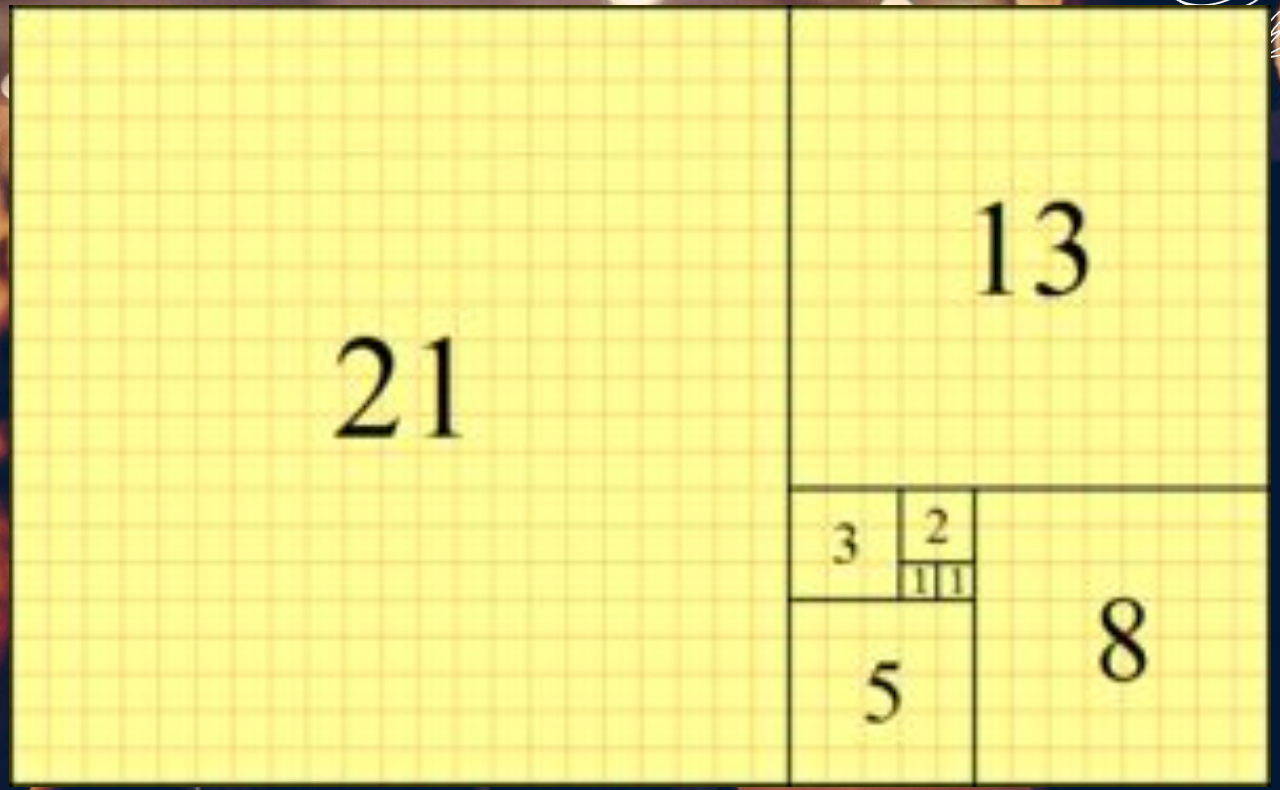
- The **Fibonacci Sequence** is the series of numbers:

  0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

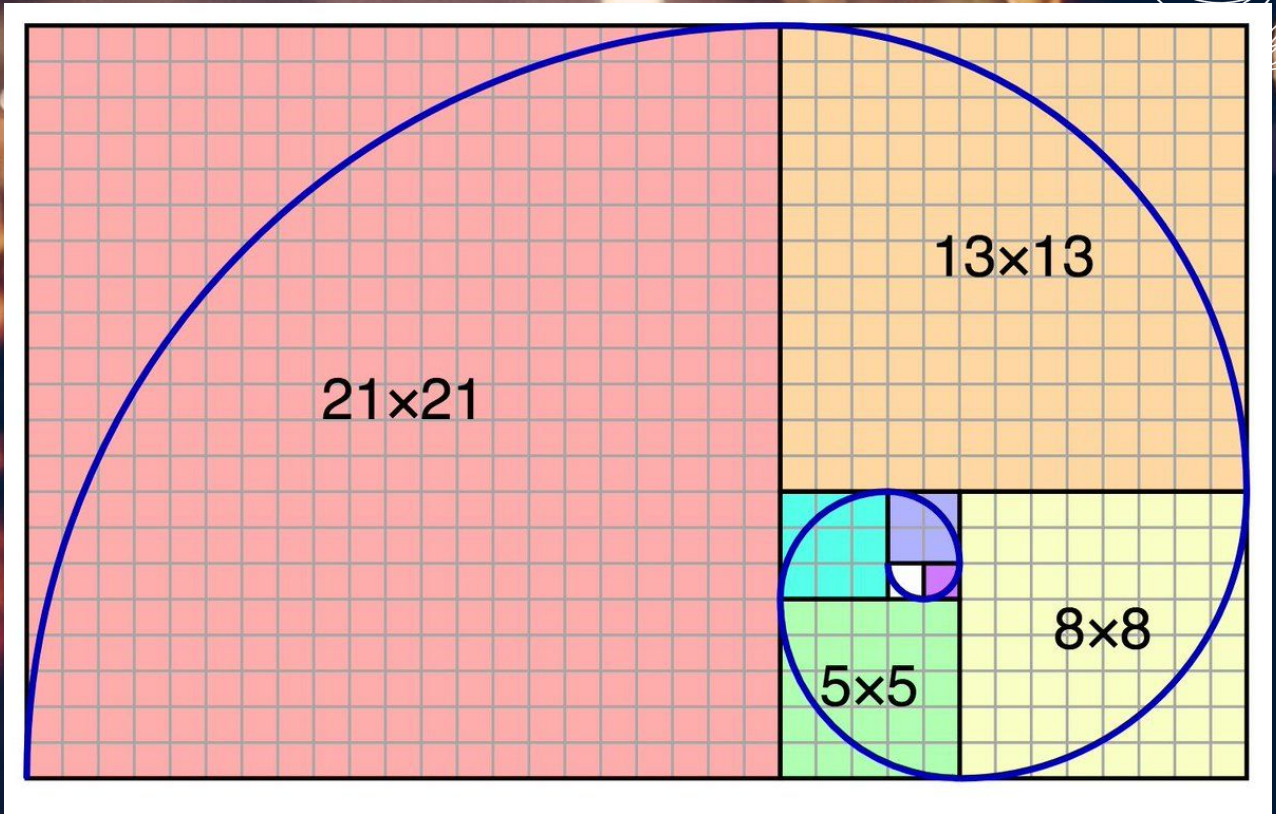
- Fibonacci is a very common sequence and appear almost everywhere!

- Let's see a few examples.

The Golden Rectangle



The Golden spiral 1



The Golden spiral 2

Who was Fibonacci?

Fibonacci, also known as Leonardo Bonacci, Leonardo of Pisa, or Leonardo Bigollo Pisano, was an Italian mathematician from the Republic of Pisa, considered to be "the most talented Western mathematician of the Middle Ages".

Sequences and terms:

A sequence is a list of numbers.

Example:

1, 3, 5, 7, 9, 11, 13, . . .

And note that a sequence can be completely random like

10002, 2, 273, 736677, 65342, 736, 289, ...

See? We are just listing out the numbers!

What are Terms ?

From the sequence 1, 3, 5, 7, 9, 11, 13, 15, . . .

What is the pattern you observe?

We say 1 is the first term.

We say 3 is the second term.

We say 7 is the fourth term.

What is the seventh term here?

Let's practice some more on Terms!

- Identify the 6th term of the sequence

  1, 1, 2, 3, 5, 8, 13, 21, 34, ...
- Identify the 7th term of the sequence

  1, 2, 6, 24, 120, 720. . . . .
- Identify the 9th term of the sequence

  1, 4, 9, 16, 25, 36, 49, 64, ...

The mathematical form of expressing a term

- We denote the first term of a sequence T_1
- The second term is T_2
- The third term is T_3
- The fourth term is T_4
- What would the tenth term be ?
- What would be the 99th term ?

Back to Fibonacci Sequence 1

- We have 1, 2, 3, 5, 8, 13, 21, 34, ...

Identify the pattern.

- So T_1 is 1, T_2 is 2,
  T_3 is 3, T_4 is 5,
  T_5 is 8, T_6 is 13,
  T_7 is 21 and so on

Back to Fibonacci Sequence 2

- As we had $1 + 2 = 3$, $2 + 3 = 5$, $3 + 5 = 8$, $5 + 8 = 13$, $8 + 13 = 21$, $13 + 21 = 34$, $21 + 34 = 55$, $34 + 55 = 89$, ...
- So we have $T_1 + T_2 = T_3$, $T_2 + T_3 = T_4$ and so on.

Final Fibonacci Sequence

The general term of Fibonacci Sequence is

$$T_{n-1} + T_n = T_{n+1}$$

Speed Round

1, 2, 3, 4, 5, 6, 7, . . .

1, 7, 13, 19, 25, 31, ….

$$1 + 6 = 7$$

$$7 + 6 = 13$$

$$13 + 6 = 19$$

$$19 + 6 = 25$$

$$25 + 6 = 31$$

$$31 + 6 = 37$$

1 , 4 , 9 , 25 , 36 , 49 , ...

1 x 1 = 1

2 x 2 = 1

3 x 3 = 9

4 x 4 = 16

5 x 5 = 25

6 x6 = 36

1, 4, 7, 10, 13,
16, 19, 22, 25,
...

$$1 + 3 = 4$$

$$4 + 3 = 7$$

$$7 + 3 = 10$$

$$10 + 3 = 13$$

$$13 + 3 = 16$$

...

25, 23, 21, 19, 17, 15, ...

25 - 2 = 23

23 - 2 = 21

21 - 2 = 19

19 - 2 = 17

17 - 2 = 15

15- 3 = 13

1, 8, 27, 64,125,216,343 ,512, ...

$1 \times 1 \times 1 = 1$

$2 \times 2 \times 2 = 8$

$3 \times 3 \times 3 = 27$

$4 \times 4 \times 4 = 64$

$5 \times 5 \times 5 = 125$

...

2, 6, 18,
54,162,486, ...

$2 \times 3 = 6$

$6 \times 3 = 18$

$18 \times 3 = 54$

$54 \times 3 = 162$

$162 \times 3 = 486$

$486 \times 3 = 1458$

1, 3, 6, 10, 15,
21, 28, 36, 45,
...

$$1 = 1$$

$$1 + 2 = 3$$

$$1 + 2 + 3 = 6$$

$$1 + 2 + 3 + 4 = 10$$

$$1 + 2 + 3 + 4 + 5 = 15$$

$$1 + 2 + 3 + 4 + 5 + 6 = 21$$

6, 8, 12, 18, 26, 36, ….

$$6 + 2 = 8$$

$$8 + 4 = 12$$

$$12 + 6 = 18$$

$$18 + 8 = 26$$

$$26 + 10 = 36$$

$$36 + 12 = 48$$

ACTIVITIES!!!!

https://mathigon.org/course/sequences/introduction

Thanks! Any questions?

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